Energy and waves

Important characteristics of a wave:

speed, frequency, wavelength, amplitude

UV light is higher in frequency than visible light; which is, in turn, higher in frequency than infrared light.

$$c = \lambda \cdot \nu$$

speed = wavelength • frequency

speed m/s = wavelength m • frequency 1/s

Three problems in Physics: Black Body Radiation, The Photoelectric Effect, and Line Spectra

Solved by doing something that had not been done before: the quantizaion of energy.

Quantization of energy in

Black Body radiation

When an object is heated it radiates energy. We have all seen and felt this phenomenon. Some radiation cannot be seen by us, but we can feel it; a wood burning stove radiates infrared light. We can feel the heat from the stove, and a simple experiment demonstrates that the heat we feel comes from light waves. To stop the heat you only have to hold an object between your face and the stove. As objects get hotter they glow like the burner of an electric stove. As objects continue to be heated they glow brilliantly like the tungsten filament in a light bulb.

Black body radiation cannot be explained using classical physics.

Max Plank saved us from the ultraviolet catastrophe by suggesting that maybe matter had to absorb energy in discrete steps. That is matter cannot absorb any amount of energy it chooses, rather matter can only absorb certain amounts of energy. The energy that could be absorbed or released is then expressed as:

$$\Delta E = n \cdot h \cdot \nu$$

(ν is the frequency of the light energy being absorbed or released)

- n is a whole number; i.e., 1,2,3,4...

- h is a constant (Planks constant 6.626 x 10^{-34} J•s which determines the size of the step).

What does this mean for our picture of oscillators...
Now an oscillator is not allowed to absorb any amount of energy. Any oscillator of a certain frequency can only have certain values as specified by $n \cdot h \cdot \nu$.

Compare a low energy oscillator, i.e. one that produces infrared light, to a high energy oscillator, one that produces ultraviolet light.

oscillator(IR) = $n \cdot (6.626 \times 10^{-34}\ J \cdot s)\ (3 \times 10^{12}\ s^{-1}) = n\ (2 \times 10^{-21}\ J)$

oscillator(UV)= $n \cdot (6.626 \times 10^{-34}\ J \cdot s)\ (3 \times 10^{16}\ s^{-1}) = n\ (2 \times 10^{-17}\ J)$

So, it takes approximately 1,000 x as much energy before UV oscillators even start going.

The photoelectric effect

Another phenomenon which classical thinking cannot explain is The Photoelectric Effect.

draw photoelectric picture with wave
table with light color, amount of current, energy of electrons

Shine white light on sodium and current flows.

Several observations were made about the photoelectric effect...

- If the light is made brighter then more current flows through the ammeter.

- If blue light is used a current flows.

- If yellow light is used current flows.

- If red light is used no current flows no mater how bright the light is.

- The electrons produced by the blue light have more energy than the electrons produced by the yellow light.

Summing up observations

- There is a frequency below which no current flows, and higher frequency light makes more energetic electrons (not more electrons).

- The amount of current, number of electrons, increases with increasing light intensity.

Can this result be explained by the classical description of light; that is, light is a wave? No.

We know that when two lights are shining on a table more energy is hitting the surface of the table. Thinking classically, if the intensity is high enough then the amount of energy being absorbed by the sodium should be enough to push an electron out, but this does not happen.

What if light was quantized; that is, what if light delivered its energy in small packets called photons.

Implications:

- The energy of the photon is determined by its frequency.
- The intensity of light is determined by the number of photons.

draw wave vs. particle (different chalk board)

The light energy is now transferred by collisions of photons with matter; in this case, the matter is an electron.

draw close-up metal

If the photon has enough energy then it collides with the electron and the electron is pushed free of the solid. This is what happens when a yellow photon hits the electron.

If a photon does not have enough energy then the electron does not gain enough energy to get free. This is what happens when a red photon hits the electron. Furthermore, if we turn up the intensity of the red light, each photon still does not have enough energy to knock an electron free. (The odds of two photons striking the electron is so small that it does not happen.)

If a blue photon hits and electron the electron will be ejected since a blue photon has more energy than a yellow photon. Since a blue photon has more energy than a yellow photon, the extra energy goes into increasing the kinetic energy of the electron.

This relationship is described mathematically:

$$\frac{1}{2} mv^2 = h\nu - \Phi$$

Φ is the work function of the metal; i.e., the amount of energy required to remove and electron.
$h\nu$ is the energy of the photon

The line spectrum of Hydrogen

Bohr created a model that accounted for the emission of specific frequencies of light from an excited hydrogen atom.

The Bohr model is derived from classical physics,

$$E = {}^{1}/_{2}mv^2 - \frac{Ze^2}{r}$$

The energy of the atom is the sum of the KE of the electron and the PE, which results from the attraction between the electron and the proton.

We do not know the radius or the velocity, but we can solve for them based on the following facts.

1. The centripetal force

$$\text{centripetal force} = \frac{mv^2}{r}$$

of the electron is counteracted by the attraction of the electron for the nucleus (Z = # of protons, e = charge of an electron, r = radius)

$$\text{Force of attraction} = \frac{Ze^2}{r^2}$$

$$\frac{Ze^2}{r^2} = \frac{mv^2}{r}$$

2. The energy of the electron must be quantized, so the angular momentum of the electron is quantized in the Bohr model.

$$mvr = \frac{n\,h}{2\pi}$$

$$E_n = -\frac{1}{2}\frac{(2\pi)^2 m e^4}{h^2}\frac{Z^2}{n^2} \quad \text{or} \quad E_n = -13.6 \text{ eV} \frac{Z^2}{n^2}$$

draw energy level diagram with E_∞, $E_{1\text{-}6}$

The Bohr model can be rewritten so that the change in energy when an electron moves from one level to another can be determined. When the Bohn equation for the energy of level n is rewritten, one gets the equation developed by J.J. Balmer ($n_f = 2$) and Johannes Rydberg

$$\Delta E = -R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

R_H is the Rydberg constant, 2.18×10^{-18} J

$E_1 = -2.18 \times 10^{-18}$ J, $E_2 = -5.45 \times 10^{-19}$ J, $E_3 = -2.42 \times 10^{-19}$ J, $E_\infty = 0$.

What is the energy and wavelength of the photon released when an electron moves from quantum level 3 to quantum level 2?

$$\Delta E = -R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\Delta E = (-2.18 \times 10^{-18})\ (1/2^2 - 1/3^2)$$

$$\Delta E = (-2.18 \times 10^{-18})\ (0.139)$$

$$\Delta E = -3.03 \times 10^{-19} \text{ J energy released}$$

$$E_{photon} = h\nu$$

$$E_{photon} = h(c/\lambda)$$

$$3.03 \times 10^{-19} \text{ J} = (6.626 \times 10^{-34})(2.9979 \text{ e } 10^8/\lambda)$$

$$\lambda = 6.56 \times 10^{-7} \text{ m}$$

$$\lambda = 656 \text{ nm (red)}$$

Transitions to level n=1 are too high in energy for us to see.

However, the Bohr model cannot be extended to model other atoms. Treating an electron as a particle failed to produce a model that can describe all the elements.

From Bohr's ideas comes De Broglie

Bohr postulated that the angular momentum of the electron was

$$\text{mvr} = \frac{n\,h}{2\pi}$$

De Broglie noted that one natural occurring phenomenon had energy level that were quantized: standing wave. So, maybe the quantization of angular momentum could be explained if the electron was considered to be a standing wave. For the "orbit" of the electron to be a standing wave, the ends of the waves must meet, or

$$2\,\pi\,\text{r} = n\,\lambda$$

Bohr's postulate can be rewritten

$$2\,\pi\,\text{r} = n\left(\frac{h}{\text{m}_\text{e}\,v}\right)$$

and from this

$$n\,\lambda = n\left(\frac{h}{\text{m}_\text{e}\,v}\right) \quad \text{or} \quad \lambda = \frac{h}{\text{m}_\text{e}\,v}$$

Yes, **electrons** (and all matter) have **wavelike** character.

X-rays and a crystal

> If a beam of X-rays is shined on a crystal, a diffraction pattern forms; if a beam of electrons is shined on a crystal a diffraction pattern forms.
>
> The crystal scatters the X-rays, and because of the regular organization of the crystal the scattered X-rays interfere with each other constructively and destructively. Since a diffraction pattern forms when a beam of electrons passes through a crystal, the electrons must be behaving like X-rays. That is the electrons are behaving like waves and interfering constructively and destructively.

A **particle**, an electron, acts like a **wave**.

This is the driving idea behind *WAVE* or *quantum* mechanics.

> The energy that a guitar string can have is quantized; that is, a guitar string can have certain wavelengths. Guitar string energy is quantized because the ends of the string are held motionless. The wavelengths that are allowed for the guitar string correspond to wavelengths that are some fraction of the length of the string.
>
> An electron can have certain wavelengths because it is trapped in an atom. The atom dictates the lowest and highest possible energy states the electron can have. The atom is like the pegs and the guitar string; the pegs determine the lowest energy possible for the wave and the string itself decides the highest energy wave—any higher and the string breaks. The electron is like the energy the guitar string can have; the energy of the electron can only go so low, just like the guitar string cannot have a

wavelength any longer than the string, and the electron can only have so much energy before it breaks free of the atom. Since the electron acts like a wave, its energy is quantized just like the energy of the guitar string.

So, the electron is a wave, does this mean that the electron is circling around the nucleus in a wavelike pattern?

fig 7.10, p 303. waves around a circle

No, as a matter of fact we have no idea what an electron is doing exactly.

To predict where the particle will go we need to know the position and the momentum (which includes directional information) of the particle.

The **Heisenberg Uncertainty Principle** states that we cannot know both the position and the momentum of a particle at a given time.

mathematically...

$$\Delta x \cdot \Delta(mv) \geq h/(4\pi)$$

$$\Delta x \cdot \Delta(mv) \geq 5.273 \text{ x } 10^{-35} \text{ J} \cdot \text{s}$$

What is the lowest possible uncertainty associated with measuring the position of a ball if the ball is traveling 10.0 m/s and the mass is 0.100 kg. The uncertainty in the speed of the ball is ±0.1, so $\Delta v=0.2$ (or 2%) The uncertainty in the measurement of the mass is ±0.001, so $\Delta m = 0.002$ (or 2%). The relative uncertainty in the momentum is the sum of the relative uncertainties of the mass and velocity. Thus, the relative uncertainty is 4% and the absolute uncertainty is $\Delta mv = 0.04$.

$$\Delta x \cdot \Delta(mv) \geq 5.273 \text{ x } 10^{-35} \text{ J} \cdot \text{s}$$

$$\Delta x \cdot 0.04 \text{ kg} \cdot \text{m} \cdot \text{s}^{-1} \geq 5.273 \text{ x } 10^{-35} \text{ J} \cdot \text{s}$$

$$\Delta x \geq 1.318 \text{ J} \cdot \text{s} \cdot \text{s}/(\text{kg} \cdot \text{m})$$

$J = kg \cdot m^2/s^2$

$$\Delta x \geq 1.318 \times 10^{-33} \frac{kg \cdot m^2 \cdot s \cdot s}{s^2 \cdot kg \cdot m}$$

$$\Delta x \geq 1.318 \times 10^{-35}\ m$$

So, when we are talking about everyday stuff we do not notice the limitation represented by the Heisenberg uncertainty principle, but on the atomic scale the error lmit is very important. Let's say for example that an electron, $m_e = 9.11 \times 10^{-31}$ kg ($\pm 0.01 \times 10^{-31}$, ± 0.11 %), is traveling at 2.19×10^6 m/s (roughly 0.7% of the speed of light, $\pm 0.01 \times 10^6$, ± 0.46 %) the uncertainty of the momentum of the electron is ($0.22 + 0.92 = 1.14\%$, $0.0114 \times 1.995 \times 10^{-24} = 2.27 \times 10^{-26}$) $2.27 \times 10^{-24}\ kg \cdot m \cdot s^{-1}$.

$$\Delta x \cdot \Delta(mv) \geq 5.273 \times 10^{-35}\ J \cdot s$$

$$\Delta x \cdot 2.27 \times 10^{-24} \geq 5.273 \times 10^{-35}\ J \cdot s$$

$$\Delta x \cdot \geq 2.32 \times 10^{-9}\ m$$

Considering an atom is approximately 10^{-10} m in diameter we cannot say where the electron is with any certainty.

So, with this large an error we cannot locate an electron and predict where the electron will go.

> Explanation is that by observing the electron we change the electron. We can use RADAR to locate a jet and we then know the jets speed and position. If we had a RADAR that could see individual electrons then you would think we could do the same thing, but we cannot. The RADAR uses light to find objects. RADAR waves go out to the plane bounce off and come back. (We get position from time, and speed from Doppler shift.) Now the RADAR does the same thing to an electron. The photon goes out collides with the electron and comes back. Now the force of impact of photon might not have been enough to move a plane, but it will be enough to move the electron. So, we might be able to find where the electron is, but not where it is going next.